Futures Markets Chapter 2: Futures Contract Pricing

11. Suppose the cost of storing gold is $6 per contract per month and $14 for a one-time transfer fee. All storage costs are paid when the gold is removed from storage. Assume that the T-Bill yield is the appropriate interest rate for an arbitrageur and is the same as in problem 10. What is the equilibrium dollar spread between the December 1988 and June 1989 gold futures prices? How does this compare with the actual spread? Express the difference as the percentage of the December 1988 futures price.

The spread costs are for a six month period, therefore total storage costs would be:

Total Per Contract Storage Costs = ($6) [6 months] + $14 [transfer fee] = $50

=========➔ $50/100 oz. = $.50 per oz.

Gold

Dec 1988 Gold Settlement price =========➔ $412.20

The equilibrium June 1989 futures price is:

$F_{t,T2} = F_{t,T1}\ (1 + r_{T1,T2}) +$ FV(storage between T1 and T2)

= $ 412.20 (1 + (6/12)(.074)) + $.50 = $427.95

The equilibrium differential is: [Equilibrium December 1988 – June 1989 Spread]

= $427.95 - $412.20 = $ 15.75

The actual differential would be:

Actual December 1988 to June 1989 Spread

= $427.80 - $412.20 = $15.60

The $.15 difference between equilibrium and actual pricing represents:

$.15/$412.20 or .04% of the contract price

13. Suppose the current spot price of copper is \$1.36 per pound. The futures price for delivery in 3 months is \$1.31 per pound. The riskless rate of interest [r_f] is 8% and the expected rate of return on the market [r_M] is 16%. The expected future spot price of copper is \$1.34 and the beta of copper is .50

a. According to the CAPM, what should be the current futures price for delivery in 3 months?

The equilibrium futures price under the CAPM should be such that :

$$E[P_T] - F_{t,T} = P_T \beta_i (r_M - r_f) \qquad (1)$$

That is, the incremental difference between the expected futures price less the equilibrium price $F_{t,T}$ is equal to the spot price [pure asset]adjusted for is risk relative to the market [$\beta_i (r_M - r_f)$]:

where $P_T = E[P_T]/ (1 + r_f + \beta_i (r_M - r_f))$ (2)

P_T = \$1.34/ (1 + .08/4 + .5(.16/4 - .08/4)) = \$1.34/(1 + .02 + .5(.04 - .02)) = \$1.34/(1.03) = \$1.30

And

$F_{t,T}$ = \$1.34 - (\$1.30) (.50)(.04 - .02) = \$1.33

Since $F_{t,T}$ = \$1.33 < the actuals futures price of \$1.31, the futures is underpriced by \$.02 per pound.

b. Can you construct a strategy using copper futures to allow for a profit from this mispricing?

Buy T-bills and then go long the futures, if you purchase P_t = \$1.30 worth of T-bills and go long one futures contract , you expected return over a 3 month period will be:

((\$1.30)(.02) + 1.34 – \$1.31)/\$1.30 = 4.31% for 3 months

[Profit from T-Bill + profit from Futures]/cost of the T-Bills

c. What will be the excess return from this strategy?

The equilibrium return is: $r_i = r_f + \beta_i (r_M - r_f)$ = .02 + .5(.04 - .02) = .03 or 3% for 3 months

Consequently, the excess return from this strategy would turn out to be:

4.31% - 3% = 1.31%